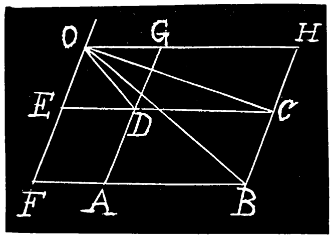
III. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let O be the given point, AB the given side. Let $OA=a$, $OB=b$, $OC=c$, $OD=d$, $AB=e$, $FA=x$, $AD=z$, $DG=y$, $\angle OFA=\theta$.



Then $a^2=(y+z)^2+x^2-2x(y+z)\cos\theta$.....(1),

$b^2=(y+z)^2+(x+e)^2-2(x+e)(y+z)\cos\theta$..........(2),

$c^2=y^2+(x+e)^2-2y(x+e)\cos\theta$.........(3),

$d^2=y^2+x^2-2xy\cos\theta$......... (4).

From (1) and (2),

$$y+z=x\cos\theta\pm\sqrt{(a^2-x^2\sin^2\theta)}=(x+e)\cos\theta \pm\sqrt{[b^2-(x+e)^2\sin^2\theta]}\text{..........}(5).$$

From (3) and (4),

$$y=(x+e)\cos\theta\pm\sqrt{[c^2-(x+e)^2\sin^2\theta]}=x\cos\theta\pm\sqrt{(d^2-x^2\sin^2\theta)}\text{.........}(6).$$

Let $4b^2e^2=A^2$, $(a^2-e^2-b^2)=B$, $2ce=C$, $d^2-e^2-c^2=D$.

Then (5) and (6) become after reduction

$$A^2-B^2=[4Be(e+x)+A^2+4e^2(e+x)^2]\sin^2\theta\text{..........}(7),$$

$$C^2-D^2=[4De(e+x)+C^2+4e^2(e+x)^2]\sin^2\theta\text{.........}(8).$$

Eliminating $\sin\theta$ we get

$$4e^2(A^2-B^2-C^2+D^2)(e+x)^2+4e[D(A^2-B^2) -B(C^2-D^2)](e+x)=B^2C^2-A^2D^2\text{.........}(9).$$

Equation (9) determines x. Then (7) or (8) determines θ, (3) or (4) determines y, (1) or (2) determines z, and the parallelogram is determined in all respects.

228. Proposed by O.E. GLENN, A.M., Fellow in Mathematics, University of Pennsylvania, Philadelphia, Pa.

Given a point O without a circle S; two arbitrary lines through O cut S in the points A, A', and B, B', respectively. Prove, by pure geometry, that the four circles through OAR, OBR, $OA'R'$, $OB'R'$, respectively, intersect in points collinear with O; R and R' being points upon S arbitrarily chosen.

No solution has been received.

CALCULUS.

179. Proposed by B. F. FINKEL, A. M., Drury College, Springfield, Mo.

Discuss the integrals of the equation $x(1-x)w''+[1-(a+b+1)x]w'-abw=0$ in the vicinities $x=0$, and $x=1$; indicating the form for the latter vicinity when $a+b=1$. Also when $1-a-b$ is an integer k. [From Forsyth's *Linear Differential Equations*, Ex. 6, p. 103].

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

This is the differential equation of the the well known hypergeometric series

$$w=1+\frac{ab}{1^2}x+\frac{a(a+1)\,b(b+1)}{1^2.2^2}x^2+\frac{a(a+1)(a+2)\,b(b+1)(b+2)}{1^2.2^2.3^2}x^3+\ldots\ldots$$

discussed in Chapter VI of Forsyth's *Differential Equations.*

If $x=0$, $w=1$; if $x>1$, w is divergent; if $x<1$, w is convergent; if $x=1$, $1-a-b=k$, a positive integer, w is convergent; if $x=1$, $1-a-b=k$, a negative integer or zero, w is divergent; if $x=1$, $a+b=1$ and w is divergent.

MECHANICS.

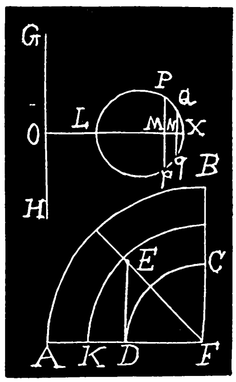
167. Proposed by EDWIN S. CRAWLEY, Ph.D., Professor of Mathematics in the University of Pennsylvania

An anchor ring or torus is produced by the revolution of a circle of radius r, the center of the revolving circle describing a circle of radius R. A quadrant of the torus is cut by two planes through the center of the ring perpendicular to each other and perpendicular to the plane of revolution. Determine the limiting value of the ratio $R:r$, so that when the quadrant thus formed is placed with one of its ends in coincidence with a horizontal plane it will rest in that position.

IV. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let the circle radius r revolve around the line GH at a distance R from its center, through an angle $2\theta=\frac{1}{2}\pi$. Let $OM=x$, $MP=y$, $MN=dx$.

The figure $PQpq$ will, by revolution, generate a quadrantal-cylindrical shell whose volume is ultimately $2y\pi xdx$. The center of gravity of this shell will be on the axis of X at a distance $\frac{x\sin\theta}{\theta}=\frac{2x\sqrt{2}}{\pi}$ from O.



$$\therefore\ \bar{x}=\frac{\int_{R-r}^{R+r}\frac{2x\sqrt{2}}{\pi}.2y\pi xdx}{\int_{R-r}^{R+r}2y\pi xdx}=\frac{2\sqrt{2}}{\pi}.\frac{\int_{R-r}^{R+r}x^2ydx}{\int_{R-r}^{R+r}xydx}$$

$$=\frac{2\sqrt{2}}{\pi}.\frac{\int_{R-r}^{R+r}x^2\sqrt{[r^2-(x-R)^2]}dx}{\int_{R-r}^{R+r}x\sqrt{[r^2-(x-R)^2]}dx}=\frac{(4R^2+r^2)\sqrt{2}}{2\pi R}.$$

$$\therefore FE=\frac{(4R^2+r^2)\sqrt{2}}{3\pi R}\text{ or }FD=\frac{4R^2+r^2}{2\pi R}.$$

Also $FK=R$, $DK=r$. By the conditions of the problem,

$$R-r=\frac{4R^2+r^2}{2\pi R}.\quad \therefore\frac{R}{r}=\frac{\pi+\sqrt{(\pi^2+2\pi-4)}}{2\pi-4}=2.9.$$